Assessment Sheet for Review/Remedial Math Tutoring

The problems below are meant to help me advise you on the best strategy for getting up to speed in your basic math skills. They are meant to be done without any external help (from either written materials or people), and without the use of a calculator. **None of the problems call for any actual numerical computations.** Keep this in mind, it might help you if you're in doubt about interpreting a problem. Take as much time as you please - this is not a speed test. Also, it is meant to be enjoyed and perhaps even learned from. Feel free to write thoughts, steps, explanations, justifications, perplexities, anything you wish. The more you write the better - it helps me see where you're at. If you find a problem intractable, please write this, too. You can either write your answers up by hand or electronically (e.g. Word or a text editor). When you're done, send the results back to me either by email or by fax. I will contact you with my comments and recommendations, which we can then discuss.

1. If I multiply a 16-digit whole number by 10,000, how many digits will the result have?

2. I have two whole numbers - one consisting of a "1" followed by 93 zeros, and the other consisting of a "1" followed by 91 zeros. If I divide the first number by the second, what will be the result? If I divide the second number by the first, what will be the result?

3. Describe succinctly the number I will get by adding together the two numbers of Exercise 2.

4. Describe succinctly the number I will get by multiplying together the two numbers of Exercise 2.

5. Describe succinctly the number I will get by subtracting the second number of Exercise 2 from the first number.

6. A certain distributor sells gold-plated pens packaged in differently-sized boxes, as follows:

A **singleton** box contains 1 pen
A **doublon** box contains 10 singleton boxes
A **triplon** box contains 10 doublons
A **quatron** box contains 10 triplons
A **quinton** box contains 10 quatrons
A **sexton** box contains 10 quintons
A **septon** box contains 10 sextons.

When somebody orders pens from this distributor, the order always gets packaged in such a way as to minimize the total number of boxes. For example, an order of 20 pens will be packaged as two doublons, and an order of 213 pens will be packaged as two triplons, a doublon, and three singletons.

(a) What is the total number of pens in a triplon?

(b) What is the total number of pens in a quinton?

(c) If the biggest-sized box used to package a given order is a triplon, and I calculate the total number of pens contained in the order, how many digits will the resulting number have?

(d) If the biggest-sized box used to package a given order is a sexton, and I calculate the total number of pens contained in the order, how many digits will the resulting number have?

(e) What is the maximum number of sextons in any one order?

(f) What is the maximum number of pens that will be packaged using boxes no bigger that a quatron?

(g) An order was shipped containing a septon, five quintons, nine triplons and six doublons. How many pens did the order contain?

(h) Somebody placed an order for one pen less than would fit in a sexton. How will the order be packaged?

(i) An order for 25,677 pens - how will it be packaged?

(j) Somebody placed two orders:
- an order for seven quatrons, five triplons, nine doublons and three singletons
- and an order for three quatrons, five triplons and two doublons.

If the two orders are combined, how will the resulting order be packaged?

(k) Somebody placed 4 separate orders, each for three quintons, five triplons and nine singletons. If the orders are combined, how will they be packaged?

7. I have two decimal fractions: one has, after the decimal point, 79 zeros followed by a "1"; the other has, after the decimal point, 75 zeros followed by a "1". If I multiply the first fraction by 10,000, what will the result look like? If I divide the second fraction by 10,000, what will the result look like?

8. Describe succinctly the number I will get if I multiply together the two fractions of Exercise 7.

9. Describe succinctly the number I will get if I divide the first fraction of Exercise 7 by the second one. And what if I divide the second fraction by the first?

10. A **palangual** of a given number is defined as a number which, when added to the given number, yields 1. For example, -15 is a palangual of 16, because

$$16 + (-15) = 1$$

(a) Are there any palanguals of -6.795? If so, give an example.

(b) Are there any palanguals of 0? If so, give an example.

(c) Does 2497^2 have any palanguals? If so, give an example.

(d) Does $(-2497)^2$ have any palanguals? If so, give an example.

(e) Does $\sqrt{18}$ have any palanguals? If so, give an example.

(f) Is it possible for a number to be its own palangual?

(g) True or false: If **a** and **b** are two numbers, and **b** is a palangual of **a**, then **a** is a palangual of **b**.

(h) Is there a number whose palangual is $\left(\frac{1}{2}\right)^{13}$? If so, give an example.

(j) Give an example of a number that has several different palanguals.

11. A **criscal** of a given number is defined as a number which, when multiplied by the given number, yields -1. For example, $-\frac{1}{3}$ is a criscal of 3, because

$$-\frac{1}{3} \cdot 3 = -1$$

(a) Does 0.25 have a criscal? If so, give an example.

(b) Does -0.025 have a criscal? If so, give an example.

(c) Does -2.39 have a criscal? If so, give an example.

(d) Does $\frac{1}{3}$ have a criscal? If so, give an example.

(e) Does -1 have a criscal? If so, give an example.

(f) Does 0 have a criscal? If so, give an example.

(g) Does $\frac{3}{4}$ have a criscal? If so, give an example.

(h) Can a number be its own criscal?

(i) True or false: A criscal of a palangual of a number is a palangual of a criscal of that number.

(j) True or false: If **a** and **b** are numbers, and you take the sum of their criscals, that sum will be a criscal of **a+b**.

(k) True or false: If **a** and **b** are numbers, and you take the product of their criscals, that product will be a criscal of **a*b**.

(l) How many numbers don't have any criscals?

(m) Give an example of a number with multiple different criscals.

12. Let **K** be defined as a completely new number - it is certainly not any of the familiar numbers! - such that $K^2 = -2$. Compute the following:

(a) $2K \cdot 3K$

(b) $\left(\sqrt{K}\right)^4$

(c) $\frac{K}{2} + \frac{1}{K}$

(d) $(3+K)(3-K)$

(e) $\frac{1}{1+K} + \frac{1}{1-K}$

(f) $\sqrt{-2}$

(g) $\frac{K}{K-2} \cdot \frac{K}{K+2}$

13. A **third-degree equation** is an equation of the form

$$ax^3 + bx^2 + cx + d = 0$$

where **a**, **b**, **c** and **d** are specific numbers. The numbers **a**, **b**, **c** and **d** are called the **coefficients** of the equation. For example,

$$6x^3 + \frac{3}{5}x^2 + 5x + 0.76 = 0$$

is a third-degree equation whose coefficients are:

$$a = 6,\ b = \frac{3}{5},\ c = 5,\ d = 0.76$$

Note that **a** is always the coefficient of x^3, **b** the coefficient of x^2, **c** the coefficient of **x** (i.e. of x^1), and **d** the constant term (i.e. the coefficient of x^0).

For each of the following third-degree equations, determine its four coefficients:

(a) $2.05x^2 + 107x^3 + x + 4 = 0$

(b) $2.05x^3 - 107x^2 + 4 = 0$

(c) $\sqrt{107}x^3 + x^3 + x - 4 = 0$

(d) $x^3 = 4$

(e) $\dfrac{2x^3}{3} = 4x$

14. The standard order of operations in an arithmetic or algebraic expression is as follows:

- Parenthesized operations take precedence over non-parenthesized operations
- Then, exponentiation and taking of roots take precedence over everything else
- Then, taking the negative takes precedence over everything else
- Then, multiplication and division take precedence over everything else
- Otherwise, all operations are performed in left-to-right order.

Thus, for example, in the expression

$$\frac{5\sqrt{12-9} + 7 \cdot 1.4 \cdot (12^{-3} - 2)}{(5+2) \cdot 12}$$

the operations will be performed in the following order:

(Step 1) Raise 12 to the power -3 and subtract 2 from the result
(Step 2) Subtract 9 from 12 ; take the square root of the result; then multiply by 5

(Step 3) Multiply together 7, 1.4, and the result of Step 1
(Step 3) Add together the results of Step 2 and Step 3
(Step 5) Add together 5 and 2, and multiply the result by 12
(Step 6) Divide the result of Step 3 by the result of Step 5

(a) Could at least some of the the operations in the above expression be performed in a different order than listed in Steps 1-6 above? If so, propose some alternatives and/or write your thoughts on this.

(b) Propose at least one correct order for performing the operations in the following expression:

$$\frac{5(\sqrt{12} - (9 + 7 \cdot 1.4) \cdot 12^{-3*2} - 2)}{5 + 2 \cdot 12}$$

15. The **distributive law** of multiplication over addition states that if **a**, **b** and **c** are any specific numbers, then

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

For example, when confronted with the problem of computing

$$7 \cdot (10 + 5)$$

(i.e. 7 times 15) we can (if we find it easier) compute, instead,

$$7 \cdot 10 + 7 \cdot 5$$

(a) How would you use the above distributive law to easily compute $17 \cdot 101$?

(b) State the above distributive law in English without using any letter-symbols to stand for numbers. (For example, the associative law for multiplication is: If **a**, **b** and **c** are any specific numbers, then **a**(**bc**) = (**ab**)**c**. One way of stating this in English might be "When taking the product of three numbers, it doesn't matter whether we first take the product of the second and third numbers and then multiply the first number by the result, or first take the product of the first and second numbers and then multiply the result by the third number".) Try to make your formulation as clear and succinct as possible.

(b) What would the distributive law of multiplication over subtraction look like? State this law and use it to easily compute $23 \cdot 99$.

(c) What would the distributive law of addition over multiplication look like? State this law. Is it in fact always true - i.e. is it really a <u>law</u>? If you think it is not always true, give an example of a specific combination of three numbers - **a**, **b** and **c** - for which it does **not** hold.